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Ideal and Optimum Cascades

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Abstract: In gaseous diffusion and gas centrifuge uranium enrichment plants, separating units are connected in cascades. The stage separation factor in gaseous diffusion is close to unity, while in a gas centrifuge it is much larger. Ideal cascades for separating binary mixtures have been designed with no mixing of the species. A new concept, particularly applicable to cascades with large stage separation factors, is called the "optimum" cascade. These cascades allow mixing, but the value of the total flow as found in some cases is less than in corresponding ideal cascades. In this paper, ideal and optimum cascades are discussed and compared.

Keywords: Binary isotope mixture, cascade, ideal cascade

INTRODUCTION

Herbst and McCandles first discussed the distinction between the no-mixing separation cascade, called the ideal cascade, and the cascade with the minimum total interstage flow (1,2). The numerical method to find parameters of the cascade with minimum total flow compared with that of an ideal one was published by Palkin (3,4). The reason for the difference of the total flows in these two cases, which sometimes can be noticeable, was suggested in a presentation of the current authors at the 8th Workshop on Separation Phenomena in Liquids and Gases held in Oak Ridge, Tennessee, USA, October 12–16, 2003. Subsequently, new data obtained in other labs (5,6) completely confirmed the conclusions

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we presented. Whereas the 8th SPLG Proceedings have been distributed only on CD for the approximately 40 workshop participants, we decided to publish our talk hereunder to reach a larger audience interested in cascade theory.

In the design of multistage installations (cascades) for uranium isotope separation the problem arises of minimization of feed flows at all cascade elements. For this purpose, K.Cohen (7,8) developed the concept of an ideal cascade. Originally, the theory of an ideal cascade was developed for the overall separation factors at cascade stages close to unity (so-called "fine" separation) that corresponded to the gaseous diffusion separation process. It has been shown that in an ideal cascade the total interstage flow is minimized. As a result, the size of the separation cascade (total number of separation elements) and the energy consumption in such a cascade are also minimized. One of the main features of an ideal cascade is the absence of thermodynamic losses from concentrations mixing in merged flows. Such cascades have been called no-mixing cascades. The theory has been broadened for the cases where the overall separation factors can vary with stage numbers (9–11) and where the total separation factors that can be much greater than unity, which is distinctive for gas centrifuges.

However, at this time, all features of ideal cascades with arbitrary stage separation factor have not been made absolutely clear. In particular, no one has been able to prove that the no-mixing condition will provide the best characteristics of an ideal cascade in which overall separation factors of the stages differ considerably from unity.

Also, note that in the classical theory of isotope separation in cascades, it is shown that in the common case the necessary and sufficient condition for cascade ideality is the following equality (7–8):

$$\alpha_s = \beta_{s+1}, s = 1, 2, \ldots N - 1, \quad (1)$$

$$\alpha_s = R'_s/R_s, \ \beta_s = R_s/R''_s, \quad (2)$$

$$R_s = C_s/1 - C_s, \quad R'_s = C'_s/1 - C'_s, \quad R''_s = C''_s/1 - C''_s \quad (3)$$

where α_s and β_{s+1} are heads and tails separation factors for a s-th and $(s+1)$-th stages, respectively; s is the current stage number; N is the total number of stages in a cascade; C_s, C'_s, C''_s are the concentrations of a key component in the feed flow to s-th stage and outlet flows from this stage enriched and depleted in a concentration of a key component, respectively.

For one particular case, an ideal cascade may be built with symmetric separation stages ($\alpha_s = \beta_s$) (9–11).

In the papers quoted above (3,4), the idea to build a cascade for uranium isotope separation with arbitrary separation factors for cascade stages that would have a total flow lower than in an ideal cascade has been published. There it was presented (and it was strange enough at the first glance) that such a cascade can be made and (what was especially unusual) it does not demand realization of the indispensable condition (1), which means assumption of no-mixing of concentrations in merged flows. This immediately raised the question about the correlation in the general case between a classical ideal cascade and the possible class of such an optimum cascade. Moreover, based on calculations presented by the author of (3,4), the claim was made that the total flow of an optimum cascade may be lower than that of an ideal one even in the presence of thermodynamic losses from concentration mixing. This resulting claim led to questioning of the mathematical model for the calculation of an optimum cascade, and in particular, the part of the model related to the transition from a system of finite-difference equations to differential equations for arbitrary separation factors at the cascade stages. The present paper is devoted to the investigation of these problems.

CLASSIFICATION FOR IDEAL CASCADES

Consider the ideal countercurrent symmetrical cascade for separation of a binary mixture of uranium isotopes comprising $N - f + 1$ separation stages in the enriching section of the cascade and $f - 1$ stages in its stripping part. The overall separation factor defined as

$$q = R'/R'' = \alpha \cdot \beta, \tag{4}$$

is identical for each cascade stage and differs considerably from unity.

Taking into account the non-mixing conditions (3) and (4), one will get the following relations

$$\begin{aligned} \alpha_1 = \beta_2 = \alpha_3 = \beta_4 = \dots \\ \beta_1 = \alpha_2 = \beta_3 = \alpha_4 = \dots \end{aligned} \tag{5}$$

Beginning calculation of the cascade with its first stage we will obtain:

$$R_1 = R_1'' \cdot \beta_1, \tag{6}$$

where $R_1'' = R^W$ is the relative concentration of the key component in the waste flow W of the cascade. For the second and the following stages of the cascade, using the relations (5) one will get:

$$\begin{aligned} R_2 &= R'_1 = \alpha_1 R_1 = \alpha_1 \beta_1 R^W = R^W q, \\ R_3 &= R'_2 = \alpha_2 R_2 = \alpha_2 q R^W = R^W q \beta_1, \\ R_4 &= R'_3 = \alpha_3 R_3 = \alpha_1 \beta_1 R^W q = R^W q^2, \\ &\dots \end{aligned} \tag{7}$$

In the general case the formulae for calculation of the relative concentrations over the cascade stages will be as follows:

$$\text{for odd stages} \quad R_s = R^W q^{\frac{s-1}{2}} \beta_1, \tag{8}$$

$$\text{for even stages} \quad R_s = R^W q^{\frac{s}{2}} \tag{9}$$

Beginning calculation of the cascade with the stage where a feed flow F enters, one can write the expressions for calculation of concentrations in product and waste flows of the cascade in terms of the concentration in the feed flow. As a result, the following relations depending on the number of stages in enriching and depleting sections of the cascade will be obtained:

$$\begin{aligned} &1type : \begin{cases} f-1 \text{ is even} : & R^P = R^F q^{\frac{N-f+1}{2}}, \\ N-f+1 \text{ is even} : & R^W = R^F q^{-\frac{f-1}{2}} \beta_1^{-1} \end{cases}, \\ &2type : \begin{cases} f-1 \text{ is even} : & R^P = R^F q^{\frac{N-f+2}{2}} \beta_1^{-1}, \\ N-f+1 \text{ is odd} : & R^W = R^F q^{-\frac{f-1}{2}} \beta_1^{-1} \end{cases}, \\ &3type : \begin{cases} f-1 \text{ is odd} : & R^P = R^F q^{\frac{N-f+1}{2}}, \\ N-f+1 \text{ is even} : & R^W = R^F q^{-\frac{f}{2}} \end{cases}, \\ &4type : \begin{cases} f-1 \text{ is odd} : & R^P = R^F q^{\frac{N-f}{2}} \beta_1, \\ N-f+1 \text{ is odd} : & R^W = R^F q^{-\frac{f}{2}}, \end{cases} \end{aligned} \tag{10}$$

where R^F, R^P, R^W are the relative concentrations of the key component in the feed F, product P, and waste W flows, respectively.

Hereafter, for convenience we will use absolute concentrations instead of the relative ones introduced above.

Combining relations (10) together with the no-mixing conditions as follows

$$C'_{s-1} = C_s = C''_{s+1}, \tag{11}$$

and the material balance in every cross section of the cascade

$$\theta_s L_S - (1-\theta_{s+1})L_{s+1} = \frac{P - for\ enricher}{-W - for\ stripper}, \tag{12}$$

one can obtain the standard system (12) for an ideal cascade given by the following equations (13)

$$\begin{cases} \delta_s' = C_{s+1} - C_s = \dfrac{\alpha_s - 1}{1 + (\alpha_s - 1)C_s} C_s(1 - C_s), \\ \theta_s = \dfrac{\beta_s - 1}{q - 1}[1 + (\alpha_s - 1)C_s], \\ L_s = \begin{cases} \frac{P(C^P - C_s)}{\theta_s \delta_s'} - for\ enricher \\ \frac{W(C_s - C^W)}{\theta_s \delta_s'} - for\ stripper \end{cases} \end{cases} \tag{13}$$

with the boundary conditions

$$\begin{cases} \theta_N L_N = P, \\ (1-\theta_1)L_1 = W. \end{cases} \tag{14}$$

This system of equations allows the no-mixing cascade to be calculated, if the value for a product flow rate from the cascade and all concentrations in the entering and outgoing flows are given beforehand.

From relations (13), it is obvious that for the case of a small concentration of a desired component and $q = const$, the cut at the first stage of the cascade θ_1 is proportional to β_1.

The analysis of the expressions obtained for the relative concentration in the cascade output flows allows two important conclusions to be drawn.

1. Concentration of a key component in product and waste flows of the cascade at fixed values of parameters N, f, q, C^F remains either constant or depends on the value of the cut at the first stage of the cascade θ_1. In the latter case the character of this dependence can be either a linear or an inversely proportional one.
2. Presence or absence of the dependence of a key-component concentration in product and waste flow rates on the cut θ_1 is determined by a relation between the number of stages in stripping and enriching sections of the cascade.

Note that as it follows from the boundary conditions (14), there is some randomness in the choice of the cut at the first stage θ_1 during calculation of the cascade. However, defining a concentration of a desired component in a feed flow to cascade, C^F, an overall separation factor at each of its separation stage, q, the number of stages in cascade, N, the number of a stage where the feed flow enters, f, and the cut at the first stage of cascade, θ_1, it is possible to calculate the distributions of all absolute concentrations and material flows over the cascade and the cuts at each of its stages. The calculations can be made by four various modes in accordance with the types of the cascades (10) depending on the relation between the number of stages in the enriching and stripping sections.

SEARCHING THE OPTIMUM PARAMETERS FOR A SYMMETRICAL COUNTERCURRENT CASCADE

To avoid errors in the transition from discrete to continuous values in the present work to optimize the cascade parameters, the numerical Nelder-Mead method of optimization was applied (13). The choice of this technique was based on the fact that it allows, without introduction of any approximations and assumptions, to find the optimum parameters of a cascade using conventional finite-difference mass transfer equations. In searching the optimum cascade parameters by the Nelder-Mead method using minimization of the total flow in a cascade, the parameters of an ideal cascade were chosen as an initial approximation.

The substance and component balance equations in the cascade stages are written as follows:

$$L_s = L'_s + L''_s, L_s C_s = L'_s C'_s + L''_s C''_s,\ s = \overline{1, N}; \tag{15}$$

which for interstage flows look as

$$L_1 = L''_2, \quad L_2 = L'_1 + L''_3, \ldots, L_f = L'_{f-1} + L''_{f+1} + F, \tag{16}$$

and finally the overall separation factors at the cascade stages usually introduce in the form as

$$\frac{C'_s}{1 - C'_s} \Big/ \frac{C''_s}{1 - C''_s} = q_s(L_s, \theta_s), \quad s = \overline{1, N}. \tag{17}$$

The equations (14)–(17) have to be accompanied with the boundary conditions

$$\begin{cases} L'' = W_1, & C''_1 = C^W; \\ L'_N = P, & C'_N = C^P. \end{cases} \tag{18}$$

In the simplest case the $q_s(L_s, \theta_s)$ functions defining the dependence of the overall separation factors from the flow entering a stage and a value of a stage cut may be assigned as a constant. In many separation processes, the overall stage separation factor depends on the cut, but here we are comparing our theory with classical theory, in which case the overall separation factor is constant. Exploring the overall separation factor varying with cut can be done in the future.

Now we introduce the auxiliary values T_s and J_s, which identify as the transit flows of the separating isotope mixture and the flow of a target component, respectively. They define the transfer of a separating substance as a whole and a target component in the direction to a product flow from a cascade and in case of the countercurrent symmetric cascade one may find them as a difference of two flows through the arbitrary cross-section between cascade stages

$$T_s = L'_{s-1} - L''_s, \tag{19}$$

$$J_s = L''_{s-1} C'_{s-1} - L''_s C''_s, \tag{20}$$

where $s = \overline{2, N}$ is a stage number on the right hand from the cross-section.

According to the balance equations for a cascade as a whole in the product (stripper) section the enriched fraction at the previous stage exceeds the depleting fraction at the next one on the value of P. For the corresponding flows of the target component this difference is PC_P. In the rectifier section the analogue values are negative: $-W$и $-WC_W$. Therefore the transit flows are equal to

$$T_s = -W, J_s = -WC^W \quad \text{for } 1 < s \leq f, \tag{21}$$

$$T_s = P, J_s = PC^P \quad \text{for } f < s \leq N. \tag{22}$$

Using the definition for overall separation factor and transit flows the following recurrent formulas to define concentration can be obtained

$$(I)\colon c'_s = \frac{qc''_s}{1 + (q-1)c''_s}, s = \overline{1, N}, \tag{23}$$

$$(II)\colon c''_s = c'_{s-1} - \frac{J_s - T_s c'_{s-1}}{L''_s}, s = \overline{2, N.} \tag{24}$$

Modification of a separating mixture composition as a result of a separation phenomenon in a stage follows from the formula (23). The second recurrent formula with the transit flows results from the balance equations in the cross-section of a cascade. It links the concentration of the enriched fraction of the previous stage with that of for depleted of the following one.

To calculate a cascade it is enough to define $N+5$ parameters: four external P, C^F, C^P, C^W and $N+1$ internal ones $N, f, L''_2, L''_3, \ldots, L''_N$. Then after determining the W and F flows and calculation of transit flows, one can find L''_1 and C''_1 from the boundary conditions for a cascade as a whole. After that using the recurrent relations (23)-(24), the concentrations in enriched and depleted fractions at all stages beginning with the first one are found. Determining the concentration C_N at the last stage finishes the calculation procedure. Finally it is necessary to check the boundary condition $C'_N = C^P$. If it is valid it means that all parameters have been defined correctly and one may pass on searching other taking interest parameters. Otherwise, one of the defined parameters should be changed and the procedure above has to be repeated.

In searching the optimum parameters of a cascade by minimization of its total flow, an initial guess can be chosen as the symmetrical ideal cascade parameters.

In the mathematical terms the search for the most efficient cascade is equivalent to searching for the minimum of the $\sum_s L_s$ functions on the set of possible values for $N, f, L_s(s = 1, 2, ..., N)$ that satisfy the following conditions

$$C'_N = \Phi(L_1, L_2, ..., L_N) = C^P, \tag{25}$$

where the function Φ represents the procedure to calculate the component concentrations over the cascade stages.

The procedure of optimization was carried out in four steps.

In the first step the number of stages N and the stage f where the feed flow enters an ideal cascade that will provide the defined concentrations in product and waste flows or ones that will be close to them are searched.

In the second step an ideal cascade with the values for N and f found in the first step are calculated. In the result, the distribution of the material flow over a cascade is defined.

Remark 1. If a few pairs for the values N and f were found, the calculation of an ideal cascade is carried out for each pair.

In the third step using the flow distribution in an ideal cascade as an initial approximation and the Nelder-Mead method, one minimizes the function of N variables, which are the feed flow rates at separation stages $L_s(1, 2, ..., N)$. The structure of this function is as follows:

$$F(L_s) = \left(\sum_s L_s\right)_{calc} + \frac{\left(\sum_s L_s\right)_{id}}{K_{id}} \left|K_{calc} - K_{def}\right|, \tag{26}$$

where $(\sum_s L_s)_{id}$ and $(\sum_s L_s)_{calc}$ are the total flows of an ideal cascade chosen as an initial approximation and an ideal cascade calculated on the current iteration, respectively.

The ratio $K_{id} = (\frac{C^P}{1-C^P} / \frac{C^W}{1-C^W})$ is the separation factor of an ideal cascade, and K_{calc}, K_{def} are the separation factors for a cascade calculated on the current iteration and for a cascade calculated by the values of concentrations of a key component in product and waste flows given beforehand, respectively.

Remark 2. The distributions of concentrations and the values for cuts over the cascade stages are calculated on each iteration.

In the fourth step the distributions of concentrations and values of cuts over stages of the cascade which have the minimum total flow among all the cascades providing the given concentrations in product and waste flows ("the optimal cascade" in the terms of the present work) are determined.

Remark 3. If the search was carried out for a few pairs of N and f, the optimal cascade among all investigated will be those for which the total flow is the minimum.

With the help of the algorithm developed, computing experiments for calculation of the optimal cascades and comparison with ideal cascades were carried out.

COMPARISON OF OPTIMUM AND IDEAL CASCADES

To compare the characteristics of optimum and ideal cascades, a series of calculations with fixed external parameters was made. Two types of cascades were chosen whose features are the most interesting from the practical point of view.

- the cascade with an even number of stages in the enriching section and odd number of stages in the stripping section (the type 2);
- the cascade with an even number of stages in the enriching and stripping sections (type 1).

The external cascade parameters were selected as follows:

1. the product flow rate to a cascade $P = 1g/s$;
2. the concentration of a key component in a feed flow $C^F = 0.711\%$;
3. the concentration of a key component in a product flow $C^P = 4.4\%$;
4. the overall separation factor at all cascade stages $q = 1.592$.

Table 1. Coincident parameters for the ideal type 2 ($N = 9$, $f = 2$) cascade of symmetrical separation elements and the optimum cascade

Stage	L, g/s	θ	C, %	C^+, %	C^-, %
1	25.14	0.443	0.56	0.71	0.45
2	45.12	0.443	0.71	0.90	0.56
3	34.10	0.443	0.90	1.13	0.71
4	25.36	0.443	1.13	1.42	0.90
5	18.42	0.444	1.42	1.78	1.13
6	12.94	0.444	1.78	2.24	1.13
7	8.52	0.445	2.24	2.81	1.42
8	5.03	0.445	2.81	3.52	2.24
9	2.24	0.446	3.52	4.40	2.81
L/P = 176.84					

The research conducted led to the conclusion that ideal and optimum cascades coincide by the total flow for all values of the overall separation factor q.

With the choice of N=9 and f=2, the minimum flow cascade was found to be the ideal cascade of type 3 while for N=10 and f=3, the minimum flow cascade was the ideal cascade of type 1 that satisfied both conditions given above. The results of the calculations are presented in Table 1 for the type 2 cascade and in Table 2 for the type 1 cascade. The numerical results for ideal and optimum cascade were identical for each of these cascade types. It was obtained that the minimum total flow in the cascades coincides with the values calculated for the ideal type 2 and 1 cascades

Table 2. Coincident parameters for the ideal type 1 ($N = 10$, $f = 3$) cascade of symmetrical separation elements and the optimum cascade

Stage	L, g/s	θ	C, %	C^+, %	C^-, %
1	18.59	0.443	0.45	0.56	0.36
2	33.37	0.443	0.56	0.71	0.45
3	45.12	0.443	0.71	0.90	0.56
4	34.10	0.443	0.90	1.13	0.71
5	25.36	0.443	1.13	1.42	0.90
6	18.42	0.444	1.42	1.78	1.13
7	12.91	0.444	1.78	2.24	1.42
8	8.52	0.445	2.24	2.81	1.78
9	5.03	0.445	2.81	3.52	2.24
10	2.24	0.446	3.52	4.40	2.81
L/P = 203.66					

Table 3. Parameters of the ideal type 1 ($N = 10$, $f = 3$) cascade of non-symmetrical separation elements

Stage	L, g/s	θ	C, %	C^+, %	C^-, %
1	57.74	0.836	0.45	0.48	0.30
2	61.51	0.110	0.48	0.71	0.45
3	96.13	0.836	0.71	0.76	0.48
4	89.23	0.110	0.76	1.13	0.71
5	54.02	0.837	1.13	1.20	0.76
6	49.67	0.110	1.20	1.78	1.13
7	27.50	0.837	1.78	1.90	1.20
8	24.75	0.111	1.90	2.81	1.78
9	10.72	0.837	2.81	2.99	1.90
10	8.98	0.111	2.99	4.40	2.81
L/P = 477.25					

($\sum L/P = 176.84$ and $\sum L/P = 203.66$ in Tables 1 and 2, respectively) and reaches their values under the cut at the first stage θ_1 corresponding to the case when all separation elements are operated in the symmetrical regime ($\alpha = \beta$).

A quite different picture was found in the case when the symmetry condition for separation elements in the ideal cascade was not kept. It is illustrated by the calculated parameters of ideal type 1 and optimum cascades constructed by the non-symmetrical separation elements with the same value of the overall separation factor as above (see the data in Tables 3 and 4, respectively).

Table 4. Parameters of the optimum cascade with the equal external parameters as above

Stage	L, g/s	θ	C, %	C^+, %	C^-, %
1	19.05	0.529	0.39	0.48	0.30
2	35.67	0.466	0.50	0.63	0.39
3	48.20	0.469	0.65	0.81	0.51
4	39.11	0.447	0.81	1.02	0.65
5	29.31	0.437	1.03	1.30	0.82
6	21.42	0.448	1.31	1.64	1.04
7	15.39	0.441	1.65	2.08	1.32
8	10.35	0.440	2.11	2.65	1.68
9	6.32	0.437	2.70	3.39	2.16
10	2.73	0.367	3.39	4.40	2.81
L/P = 277.50					

In this example the total flow of the ideal type 1 cascade exceeds that of for the optimum one more than 50%. It is evidence that in the cascade constructed of non-symmetric elements the non-mixing condition $\alpha_s = \beta_{s+1}$ does not coincide with the condition of the minimum total flow.

The comparison of the results obtained in the present work with the data of (3) allowed the authors to come to the conclusion that both approaches give very close results for a large number of stages in a cascade and overall separation factors close to unity. In the case of The so-called "short" cascades and overall separation factors that differ considerably from unity, the approach from (3) gives appreciable mistakes because of the application of differential mathematics to a discrete object.

THE OPTIMUM CUT OF SEPARATING ELEMENT IN OPTIMUM AND IDEAL CASCADES

As is known, the optimum cut providing the maximum value of the separation power for cascade stages operating in the non-symmetrical regime with an overall separation factor that differs substantially from unity in the case of $C << 1$ is defined as (10)

$$\theta_{opt} = \frac{1}{\ln q} - \frac{1}{q-1}. \tag{27}$$

However, the cut ensuring the minimum total flow in an ideal cascade with symmetrical separation elements ($\alpha = \beta$) depends on the value of an overall separation factor in the following way (7)

$$\theta_{id} = \frac{1}{\sqrt{q}+1}. \tag{28}$$

The corresponding dependences for θ_{opt} and θ_{id} versus a value of an overall separation factor at cascade stages are shown in Fig. 1.

The distinction in the values for θ_{opt} and θ_{id} is explained as follows. When an overall separation factor at cascade stages q is close to unity, the condition of the optimum operation of separation element and the non-mixing condition practically coincide either in the case of symmetrical or non-symmetrical elements. For overall separation factors q that differ considerably from unity, in the case of the cascade made of symmetrical elements the condition of ideality prevails over the condition of the optimum operation for a single separation element. It leads to the situation when the total flows in ideal and optimum cascades coincide. In contrast, in the case of non-symmetrically operated elements, the condition of the optimum work of a single separation unit is much more important than the non-mixing condition. It defines the situation when

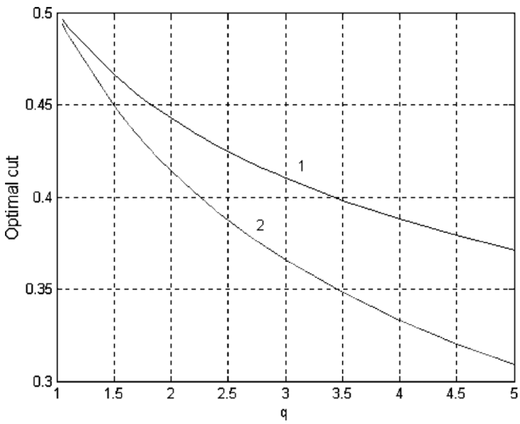

Figure 1. Dependence of the optimum cut versus the value of the overall separation factor for 1-optimum and 2–ideal cascades.

the total flow in an ideal cascade will be less than in the case of symmetrically operated elements. As is evident from the dependences shown in Fig. 1, this difference increases with growth of the magnitude of q. This distinction is defined by the fact that connection of non-symmetrically operated elements in a non-symmetrical scheme of a cascade leads to strongly non-optimum conditions of elements exploitation, which are operating with substantial underloading.

The conclusion made is confirmed by calculation of the separation powers of stages in the ideal and optimum cascades. The calculations are carried out for the external parameters given in Section 4.

Table 5. Parameters of the ideal type 3 cascade ($N = 9, f = 3$) of non-symmetrical separation elements

L, g/s	θ	C, %	C^+, %	C^-, %	δU, g/s	$(\delta U/L) \cdot 10^2$
1.40	0.012	0.45	0.706	0.44	0.0207	1.48
75.51	0.981	0.706	0.71	0.45	1.2935	1.71
76.39	0.118	0.71	1.12	0.706	1.1329	1.48
43.22	0.982	1.12	1.13	0.71	0.7390	1.71
42.82	0.118	1.13	1.78	1.12	0.6364	1.48
21.97	0.982	1.78	1.79	1.13	0.3748	1.70
21.72	0.118	1.79	2.80	1.78	0.3241	1.49
8.56	0.982	2.80	2.82	1.79	0.1454	1.70
8.40	0.119	2.82	4.40	2.80	0.1261	1.50
$\sum$ L/P = 299.99						

Table 6. Parameters of the corresponding optimum cascade with the equal external parameters as in the ideal type 3 cascade

L, g/s	θ	C, %	C^{+}, %	C^{-}, %	δU, g/s	$(\delta U/L) \cdot 10^2$
19.74	0.300	0.52	0.71	0.44	0.4775	2.42
33.09	0.404	0.65	0.83	0.52	0.8868	2.68
44.87	0.395	0.78	1.01	0.63	1.1967	2.67
29.97	0.443	1.01	1.28	0.81	0.8131	2.71
21.99	0.442	1.29	1.62	1.02	0.5966	2.71
15.64	0.442	1.64	2.06	1.30	0.4242	2.71
10.55	0.440	2.09	2.63	1.67	0.2860	2.71
6.41	0.432	2.68	3.38	2.15	0.1735	2.71
2.77	0.361	3.38	4.40	2.80	0.0721	2.60
$\sum$ L/P = 185.03						

For the ideal type 2 cascade of ($N = 9$ and $f = 3$), the following set of parameters satisfying the given external conditions were obtained and presented in Table 5.

The sum of the separation powers of all cascade stages is equal to $\sum \delta U = 4.793$ g/s and the sum of the specific separation powers is equal to $\sum \delta U/L = 1.425 \cdot 10^{-1}$.

The procedure of optimization that led to the parameters of the optimum cascade is presented in Table 6.

The sum of the separation powers of all cascade stages is equal to $\sum \delta U = 4.926$g/s, which is only a bit higher than in the ideal cascade. However, the sum of the specific separation powers of all cascade stages is equal to $\sum \delta U/L = 2.392 \cdot 10^{-1}$, which is noticeably higher than in the ideal cascade.

Analyzing the results obtained, one can compute the cascade efficiency coefficient $\eta = (\sum \delta U)_{id}/(\sum \delta U)_{opt} = 0.973$ that takes into account non-ideality of a cascade profile and mixing losses connected with it. We can also compute the quantity $\eta(\sum \delta U/L)_{opt}/(\sum \delta U/L)_{id} = 1.63$. Therefore, we have shown that despite the fact that the cascade efficiency coefficient is less than unity, the sum of the specific separation powers in an optimum cascade becomes significantly higher in comparison with that for ideal one.

It means that all separation stages in the optimum cascade work more effectively and therefore, the total flow in such a cascade becomes lower considerably in contrast to the ideal one. Besides, comparing the cut values in both cascades under investigation, one can see that in the optimal cascade they are more attractive from a technological point of view because of slightly varying over cascade stages.

Recently it has been demonstrated that the value of a total flow in the ideal cascade with symmetric elements and the overall separation factors higher than 5–6 is greater that in the optimum cascade (14). It is explained by the considerable difference of the cut values in the ideal and optimum cascades for the large overall separation factors (see Fig. 1).

CONCLUSIONS

1. The classification for ideal cascades with fixed values of N, f, q, C^F, C^P is developed. Depending on the character of variation of the concentration of a key component in product and waste flows on the cut at the first stage of a cascade θ_1, all ideal cascades are divided into four groups characterizing various relations between numbers of stages in enriching and stripping sections of a cascade.
2. It is found that for all four groups of ideal cascades the optimum value of θ_1 is introduced ensuring the minimum total flow in a cascade exists.
3. It is shown that the total flow in a cascade, in which the condition $\alpha = \beta = \sqrt{q}$ is valid for all separation elements, coincides with the total flow in an ideal cascade for arbitrary values of q close to unity. For overall separation factors considerably higher than unity, the distinction between ideal and optimal cascade is essential.
4. It is established that in the case of symmetrical separation elements, the non-mixing condition prevails over the condition of optimum (from the point of view the separation power) operating work of a single separation element. Under this condition, the concepts of ideal and optimum cascades coincide. For the cascade of non-symmetrical elements, the condition of the optimum work of a single separation element prevails over the non-mixing condition that leads to divergence in the values for the total flows in ideal and optimum cascades that increase with growth of the value for the overall separation factor q. This explains the well-known practical rule according to which non-symmetrical separation stages with big overall separation factors are preferable to connect by a non-symmetric scheme.

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